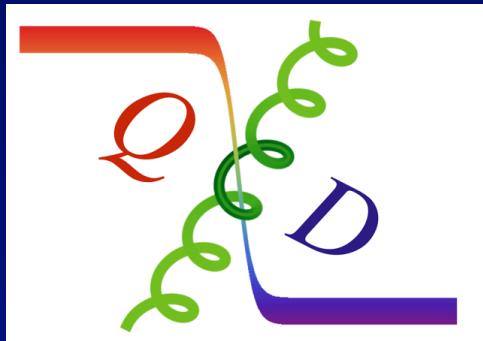


PDF from Hadronic Tensor on the Lattice and Connected Sea Evolution

- Path-integral Formulation of Hadronic Tensor in DIS
- Parton Degrees of Freedom
- Numerical Challenges
- Evolution of Connected Sea Partons
- Quasi-PDF

χ QCD Collaboration



POETIC, Temple U., Nov. 14, 2016

Experimental Data

- New Muon Collaboration (NMC – PRL 66, 2712 (1991)) $\mu + p(n) \rightarrow \mu X$

$$S_G(x_0, x_1; Q^2) = \int_{x_0}^{x_1} dx \frac{F_2^{\mu p}(x, Q^2) - F_2^{\mu n}(x, Q^2)}{x}$$

Quark parton model + Isospin symmetry

$$S_G(0,1; Q^2) = \frac{1}{3} + \frac{2}{3} \int_0^1 dx (\bar{u}_p(x) - \bar{d}_p(x)); \quad S_G(0,1; Q^2) = \frac{1}{3} \text{(Gottfried Sum Rule)}$$

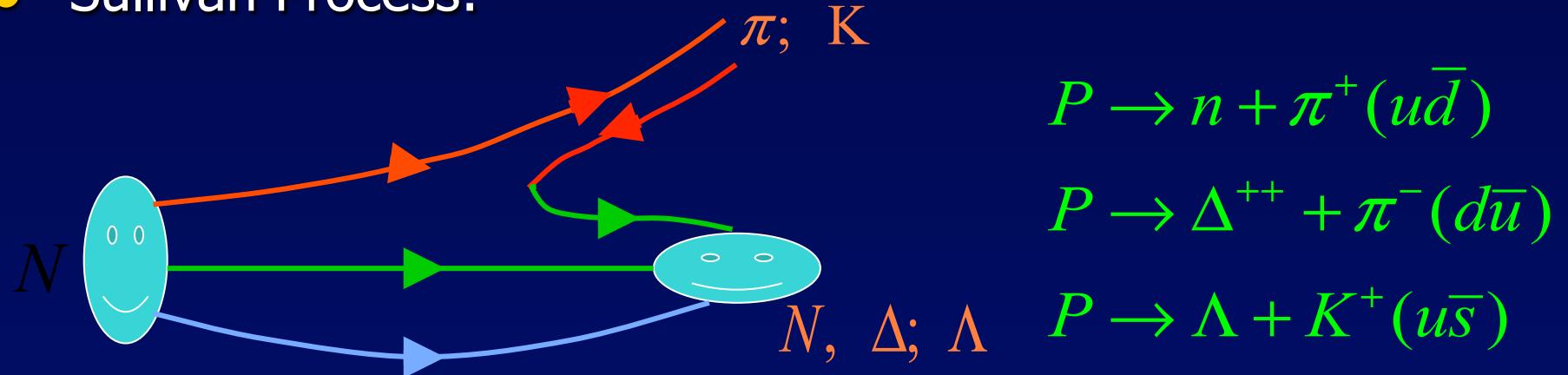
NMC : $S_G(0,1; 4 \text{ GeV}^2) = 0.240 \pm 0.016$ (5σ from GSR)

- \bar{d}/\bar{u} asymmetry from Drell-Yan Production (PRL 69, 1726 (1992))
- NuTeV experiment (PRL 88, 091802 (2002))

$$\sin^2 \theta_w \text{ (3}\sigma \text{ from Standard Model)} \xrightarrow{?} s(x) \neq \bar{s}(x)$$

Possible Explanation

- Perturbative QCD: $g \rightarrow u\bar{u} = g \rightarrow d\bar{d}$
Higher order effects on $\bar{u}_P \neq \bar{d}_P$ is small.
- Sullivan Process:



- Need a non-perturbative formulation to reveal $\bar{u}_P \neq \bar{d}_P$ in QCD and a scheme to calculate it quantitatively \longrightarrow Euclidean path-integral formalism and lattice gauge calculation.

Hadronic Tensor in Euclidean Path-Integral Formalism

- Deep inelastic scattering
In Minkowski space

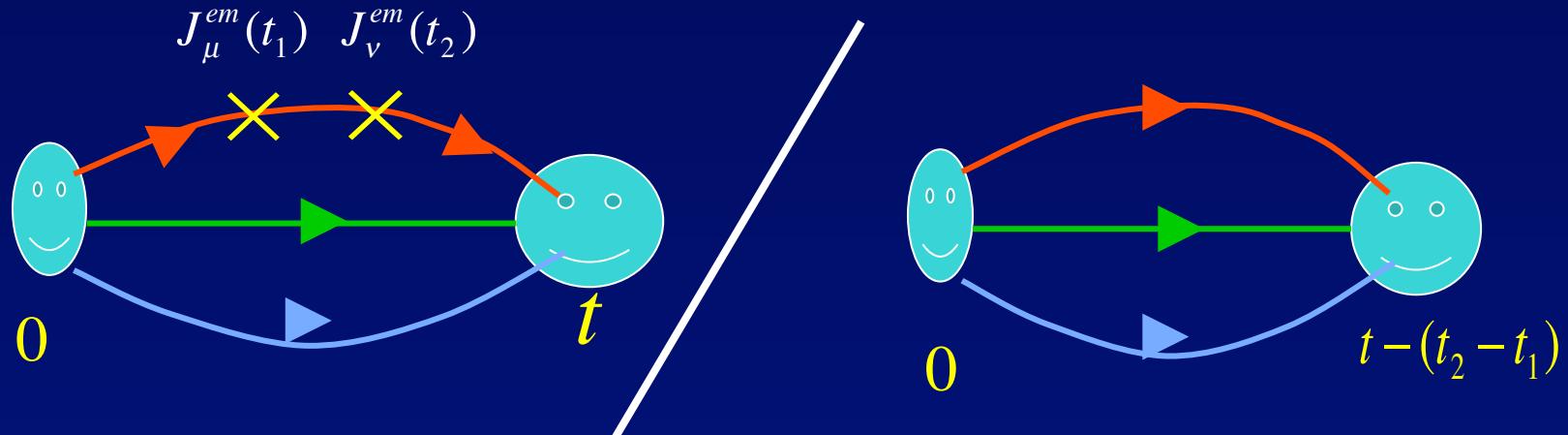
$$\frac{d^2\sigma}{dE' d\Omega} = \frac{\alpha^2}{q^4} \left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu}(\vec{q}, \vec{p}, v) = \frac{1}{\pi} \text{Im} T_{\mu\nu} = \langle N(\vec{p}) | \int \frac{d^4x}{4\pi} e^{iq \cdot x} J_\mu(x) J_\nu(0) | N(\vec{p}) \rangle_{\text{spin avg}}$$

$$= \frac{1}{2} \sum_n \int \prod_{i=1}^n \left[\frac{d^3 p_i}{(2\pi)^3 2E_{pi}} \right] (2\pi)^3 \delta^4(p_n - p - q) \langle N(\vec{p}) | J_\mu | n \rangle \langle n | J_\nu | N(\vec{p}) \rangle_{\text{spin avg}}$$

- Euclidean path-integral

K.F. Liu, PRD 62, 074501 (2000)



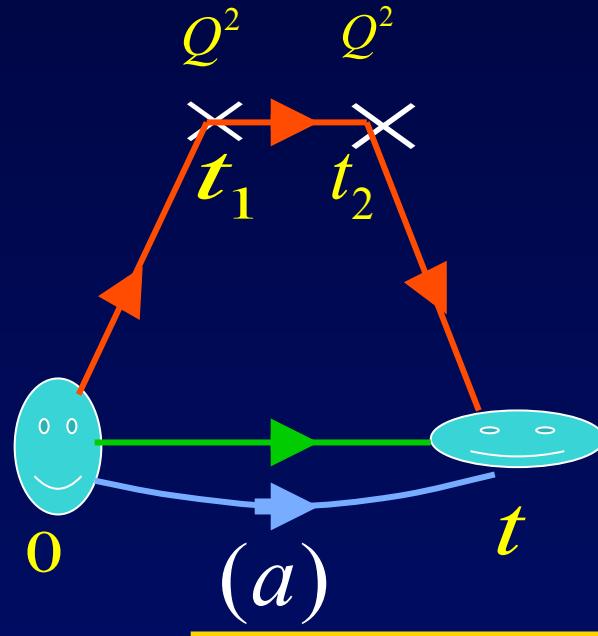
$W_{\mu\nu}$ in Euclidean Space

$$\begin{aligned}
 \tilde{W}_{\mu\nu}(\vec{q}, \vec{p}, \tau = t_2 - t_1) &= \frac{\frac{E_P}{M_N} \text{Tr} \langle \Gamma_e \chi_N(\vec{p}, t) \sum_{\vec{x}} \frac{1}{4\pi} e^{-i\vec{q}\cdot\vec{x}} J_\mu(\vec{x}, t_2) J_\nu(0, t_1) \chi_N^\dagger(\vec{p}, 0) \rangle}{\text{Tr} \langle \Gamma_e \chi_N(\vec{p}, t) \chi_N^\dagger(\vec{p}, 0) \rangle} \\
 &\xrightarrow[t=t_2 \gg l/\Delta E_P, t_1 \gg l/\Delta E_P]{} \\
 &= \frac{1}{4\pi} \sum_n \left(\frac{2m_N}{2E_n} \right) \delta_{\vec{p}_n - \vec{p} - \vec{q}} \langle N(\vec{p}) | J_\mu | n \rangle \langle n | J_\nu | N(\vec{p}) \rangle_{\text{spin avg}} e^{-(E_n - E_P)\tau} \\
 &= \langle N(\vec{p}) | \sum_{\vec{x}} \frac{e^{-i\vec{q}\cdot\vec{x}}}{4\pi} J_\mu(\vec{x}, \tau) J_\nu(0, 0) | N(\vec{p}) \rangle_{\text{spin avg}}
 \end{aligned}$$

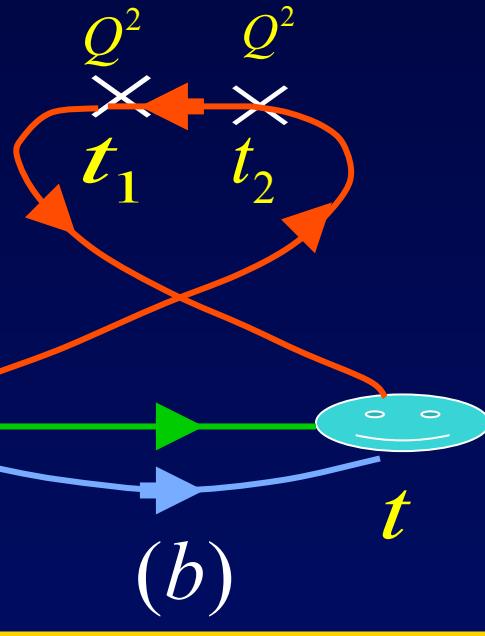
Laplace transform

$$W_{\mu\nu}(\vec{q}, \vec{p}, v) = \frac{1}{i} \int_{c-i\infty}^{c+i\infty} d\tau e^{v\tau} \tilde{W}_{\mu\nu}(\vec{q}, \vec{p}, \tau)$$

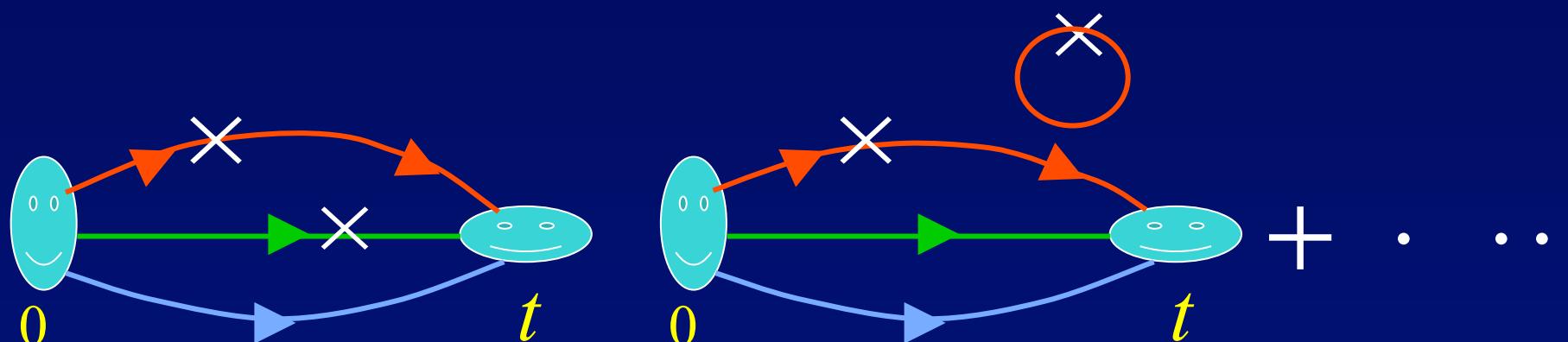
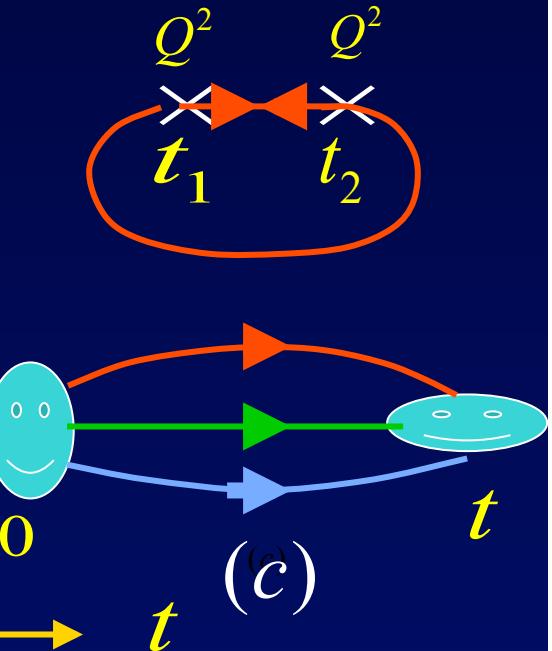
$$q = q_V + q_{CS}$$



$$\bar{q}_{CS}$$



$$q_{DS} = (\neq ?) \bar{q}_{DS}$$



Cat's ears diagrams are suppressed by $O(1/Q^2)$.

- $W_{\mu\nu}(p, q) = -W_1(q^2, v)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + W_2(q^2, v)(p_\mu - \frac{p \cdot q}{q^2} q_\mu)(p_\nu - \frac{p \cdot q}{q^2} q_\nu)$

- Bjorken limits

$$vW_2(q^2, v) \longrightarrow F_2(x, Q^2) = x \sum_i e_i^2 (q_i(x, Q^2) + \bar{q}_i(x, Q^2)); \quad x = \frac{Q^2}{2 p \cdot q}$$

- Parton degrees of freedom: valence, connected sea and disconnected sea

u	d	s
$u_V(x) + u_{CS}(x)$	$d_V(x) + d_{CS}(x)$	
$\bar{u}_{CS}(x)$	$\bar{d}_{CS}(x)$	
$u_{DS}(x) + \bar{u}_{DS}(x)$	$d_{DS}(x) + \bar{d}_{DS}(x)$	$s_{DS}(x) + \bar{s}_{DS}(x)$

Properties of this separation

- Gauge invariant
- Topologically distinct as far as the quark lines are concerned
- Structure functions W_1 and W_2 are frame independent.
- Small x behavior of CS and DS are different.

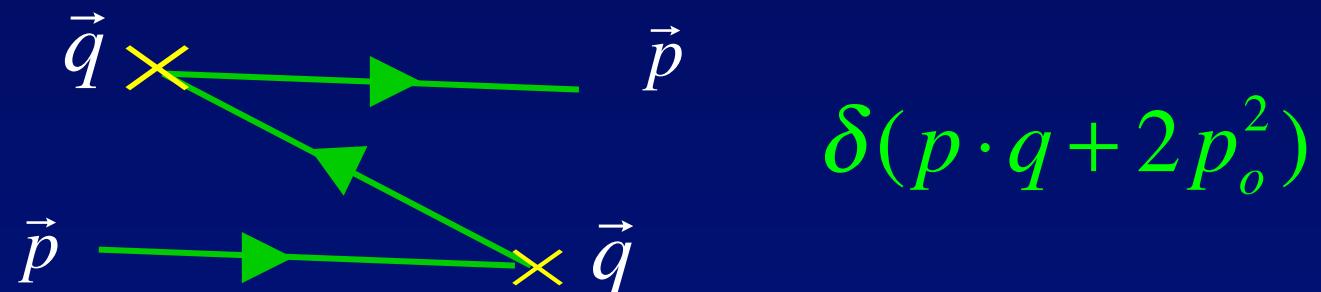
$$q_V, q_{CS}, \bar{q}_{CS} \sim_{x \rightarrow 0} x^{-\alpha_R} (x^{-1/2})$$

$$q_{DS}, \bar{q}_{DS} \sim_{x \rightarrow 0} x^{-1}$$

- Note that diagram (b) are from pre-existing connected sea antipartons the same way as in (c) which involves pre-existing disconnected sea partons and antipartons.



- Whereas, current induced pair productions are suppressed as $O(\vec{q}^2 / \vec{p}^2)$.



Operator Product Expansion -> Taylor Expansion

- Operator product expansion

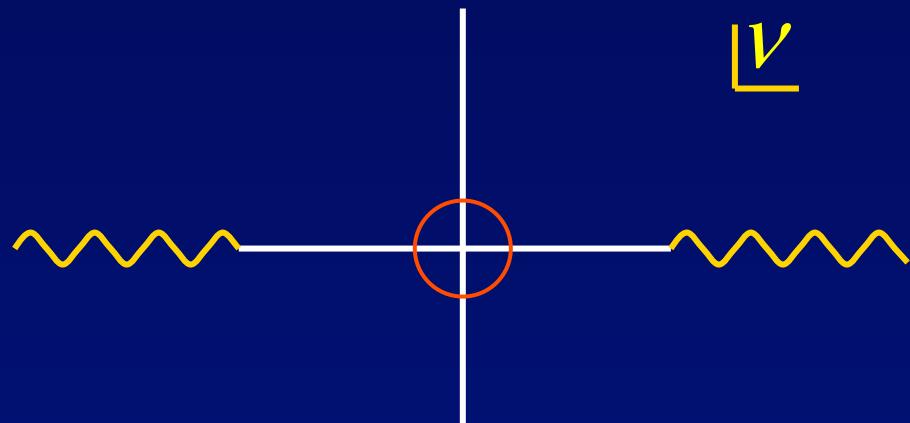
$$W_{\mu\nu} = \frac{1}{\pi} \operatorname{Im} T_{\mu\nu}$$

- Dispersion relation

$$T_{\mu\nu} = \frac{1}{\pi} \int_{Q^2/2M_N}^{\infty} d\nu' \frac{\nu' W_{\mu\nu}(q^2, \nu')}{\nu'^2 - \nu^2}$$

- Expand in the unphysical region

$$\frac{2M_N \nu}{Q^2} = \frac{2p \cdot q}{Q^2} < 1 \quad (\text{x} > 1)$$



- Euclidean path-integral

- Consider

$$\tilde{W}_{\mu\nu}(q^2, \tau)|_{(a)} \propto \int D[A] \det M(A) e^{-S_g}$$

$$\times Tr \left[\dots M^{-1}(t, t_2) \int d^3x e^{-i\vec{q}\cdot\vec{x}} i\gamma_\mu M^{-1}(t_2, t_2 - \tau) i\gamma_\nu M^{-1}(t_2 - \tau, 0) \dots \right]$$

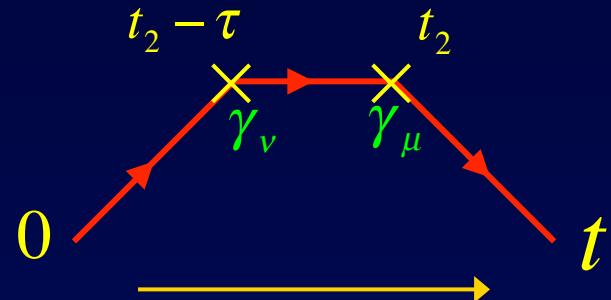
- Short-distance expansion ($|\vec{x}|, \tau \rightarrow 0$ from $|\vec{q}|, v \rightarrow \infty$)

$$M^{-1}(t_2, t_2 - \tau) \xrightarrow{\text{free quark}} \frac{1}{4\pi^2} \frac{\partial'}{\vec{x}^2 + \tau^2};$$

$$M^{-1}(t_2 - \tau, 0) \xrightarrow{|\vec{x}|, \tau \rightarrow 0} e^{\vec{x} \cdot \vec{D} + \tau D_\tau} M^{-1}(t_2, 0)$$

- Laplace transform

$$W_{\mu\nu}(q^2, v) \propto Tr \left[\dots M^{-1}(t, t_2) i\gamma_\mu \frac{-i\pi(q + i\vec{p})}{|\vec{q} + i\vec{D}|} \delta(v + D_\tau - |\vec{q} + i\vec{D}|) i\gamma_\nu M^{-1}(t_2, 0) \dots \right]$$



- Dispersion relation

$$T_{\mu\nu}(q^2, v) = \frac{1}{\pi} \int_{Q^2/2M_N + D_\tau}^{\infty} dv' \frac{v' W_{\mu\nu}(q^2, v' - D_\tau)}{v'^2 - (v + D_\tau)^2},$$

$$\propto \text{Tr} \left[\dots M^{-1}(t, t_2) i\gamma_\mu \frac{-i(\not{q} + i\not{D})}{(Q^2 + 2iq \cdot D - D^2)} i\gamma_\nu M^{-1}(t_2, 0) \dots \right],$$

where $\tau = it$ and $D_t = iD_\tau$

so that $D = (\vec{D}, -iD_t)$ is covariant derivative in Minkowski space.

- Expansion about the unphysical region ($2q \cdot p / Q^2 < 1$)

$$T_{\mu\nu}(q_V + q_{CS}) = \sum_f e_f^2 \left[8 p_\mu p_\nu \sum_{n=2} \frac{(-2q \cdot p)^{n-2}}{(Q^2)^{n-1}} A_f^n(CI) - 2 g_{\mu\nu} \sum_{n=2} \frac{(-2q \cdot p)^n}{(Q^2)^n} A_f^n(CI) \right]$$

even + odd n terms

- $A_f^n = ?$

$$A_f^n(CI) \propto \int D[A] \det M(A) e^{-S_g} \text{Tr} \left[\dots M^{-1}(t, t_2) O_f^n M^{-1}(t_2, 0) \dots \right]$$

$$O_f^n = i\gamma_{\mu_1} \left(\frac{-i}{2}\right)^{n-1} \vec{D}_{\mu_2} \vec{D}_{\mu_3} \dots \vec{D}_{\mu_n},$$

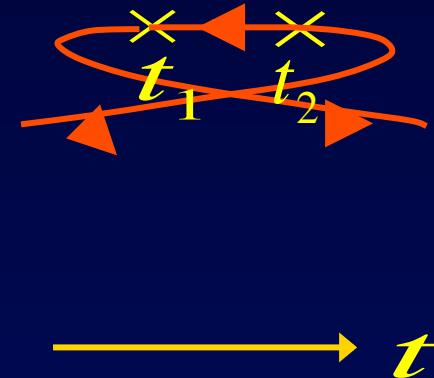
$$\langle p | \bar{\psi} | O_f^n | \psi | p \rangle = A_f^n(CI) 2 p_{\mu_1} p_{\mu_2} \dots p_{\mu_n}$$

12

- Similarly for \bar{q}_{CS}
except with $q \rightarrow -q$

$$T_{\mu\nu}(\bar{q}_{CS}) = \sum_{even, n=2} ... A_f^n(CI) - \sum_{odd, n=3} ... A_f^n(CI)$$

even – odd



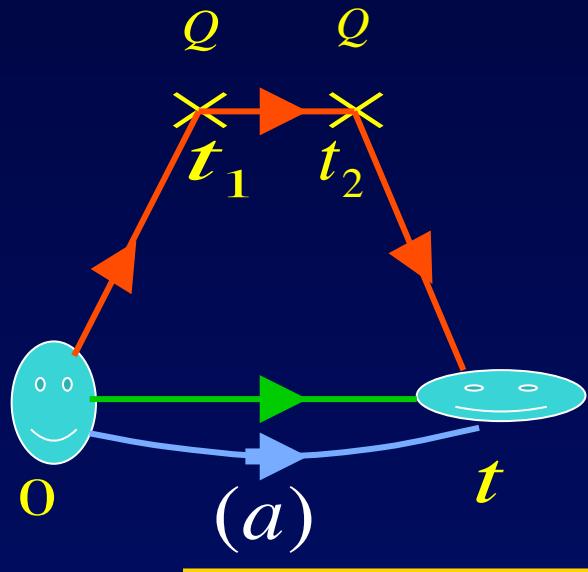
- For q_{DS} / \bar{q}_{DS}

$$T_{\mu\nu}(q_{DS} / \bar{q}_{DS}) = \sum_{even, n=2} ... A_f^n(DI) \pm \sum_{odd, n=3} ... A_f^n(DI)$$

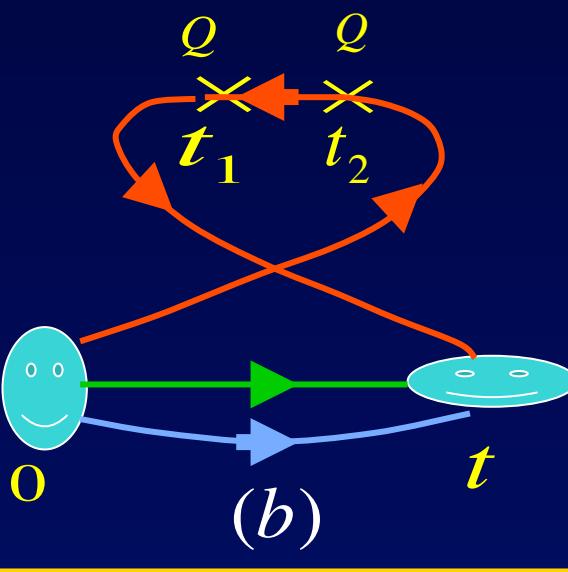
- DIS with electromagnetic currents J_μ^{em}

$$\begin{aligned} T_{\mu\nu} &= T_{\mu\nu}(q_V + q_{CS}) + T_{\mu\nu}(\bar{q}_{CS}) + T_{\mu\nu}(q_{DS}) + T_{\mu\nu}(\bar{q}_{DS}), \\ &= 2 \sum_{even, n=2} ... [A_f^n(CI) + A_f^n(DI)] \end{aligned}$$

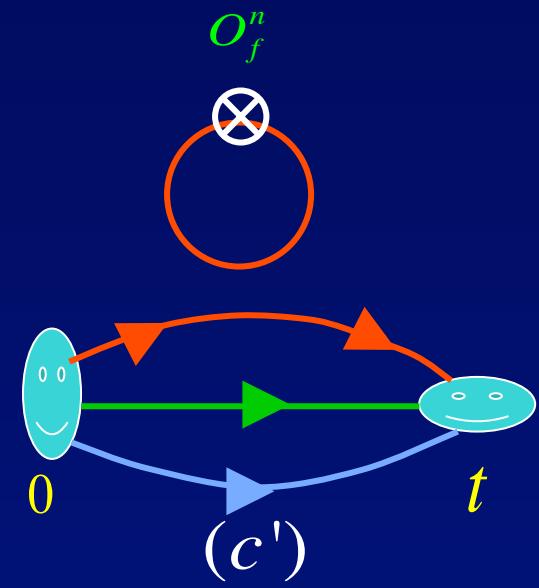
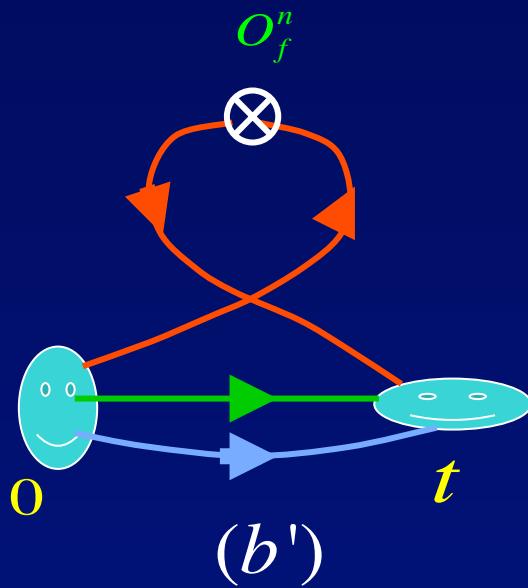
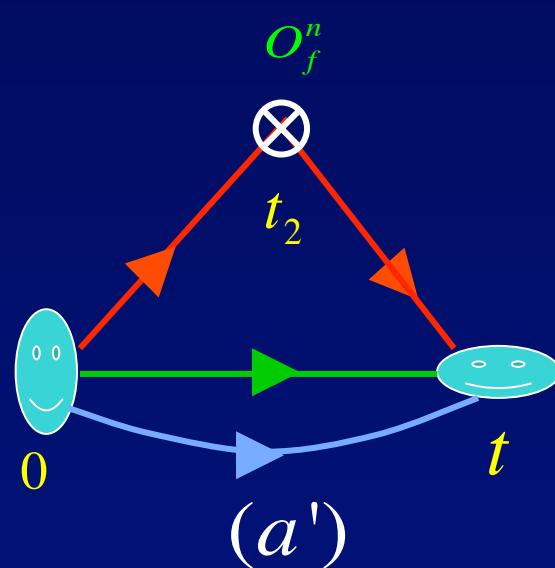
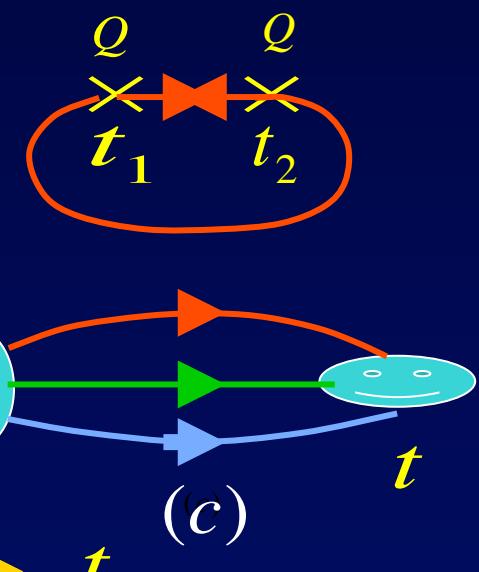
$$q = q_V + q_{CS}$$



$$\bar{q}_{CS}$$



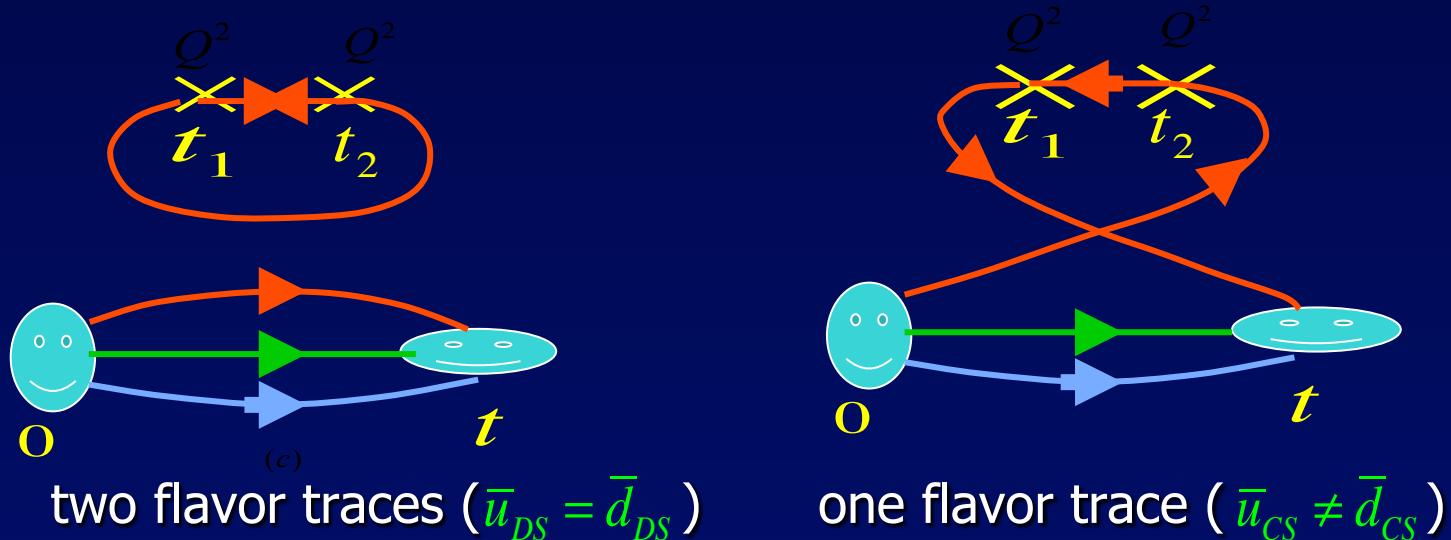
$$q_{DS} = (\neq ?) \bar{q}_{DS}$$



Gottfried Sum Rule Violation

$$S_G(0,1;Q^2) = \frac{1}{3} + \frac{2}{3} \int_0^1 dx (\bar{u}_p(x) - \bar{d}_p(x)); \quad S_G(0,1;Q^2) = \frac{1}{3} \text{ (Gottfried Sum Rule)}$$

NMC: $S_G(0,1;4 \text{ GeV}^2) = 0.240 \pm 0.016$ (5σ from GSR)



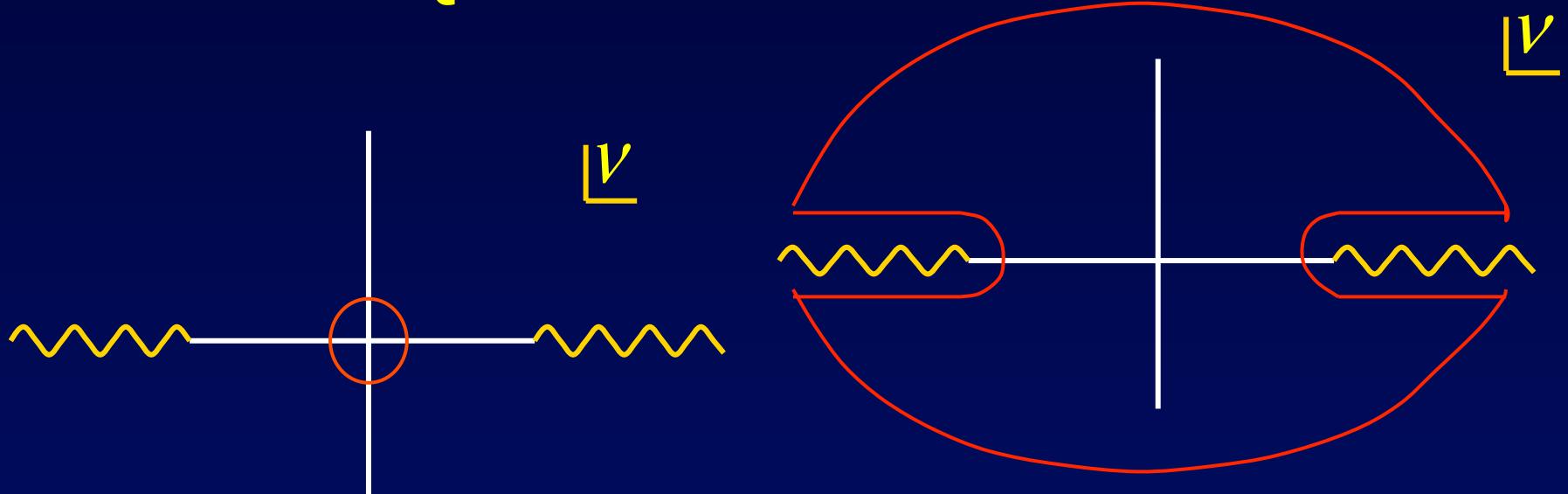
K.F. Liu and S.J. Dong, PRL 72, 1790 (1994)

$$\begin{aligned} \text{Sum} &= \frac{1}{3} + \frac{2}{3} \int_0^1 dx (\bar{u}_{CS}(x) - \bar{d}_{CS}(x)), \\ &= \frac{1}{3} + \frac{2}{3} [n_{\bar{u}_{CS}} - n_{\bar{d}_{CS}}] (1 + O(\alpha_s^2)) \end{aligned}$$

Comments

- *The results are the same as derived from the conventional operator product expansion.*
- *The OPE turns out to be Taylor expansion of functions in the path-integral formalism.*
- *Contrary to conventional OPE, the path-integral formalism admits separation of CI and DI.*
- *For O_f^n with definite n , there is only one CI and one DI in the three-point function, i.e. (a') is the same as (b'). Thus, one cannot separate quark contribution from that of antiquark in matrix elements.*

Quark Parton Model



$$I_n = \int \frac{dv}{2\pi i} \frac{1}{v^{n-1}} T_2(Q^2, v),$$

$$= \sum_f 8 e_f^2 \left(\frac{2M_N}{Q^2} \right)^{n-1} A_f^n$$

$$I_n = 2 \int_{Q^2}^{\infty} \frac{dv}{2\pi i} \frac{2i}{v^{n-1}} W_2(Q^2, v),$$

$$= 8 \left(\frac{2M_N}{Q^2} \right)^{n-1} \int_0^1 dx x^{n-2} \frac{2M_N v W_2(Q^2, v)}{4}$$

- $A_f^{n=even}(CI) \equiv M_f^n(CI) = \int_0^1 dx x^{n-1} (q_V(x) + q_{CS}(x) + \bar{q}_{CS}(x))_f$

- $A_f^{n=odd}(CI) \equiv M_f^n(CI) = \int_0^1 dx x^{n-1} q_V(x)_f$

- $A_f^{n=even}(DI) \equiv M_f^n(DI) = \int_0^1 dx x^{n-1} (q_{DS}(x) + \bar{q}_{DS}(x))_f$

Operator Mixing

- Connected insertion

$$\frac{d M_f^n(CI)}{d \log Q^2} = \frac{a_f^n}{2b_0} \frac{1}{\log(Q^2 / \Lambda^2)} M_f^n(CI)$$

- Disconnected insertion

$$\frac{d M_f^n(DI)}{d \log Q^2} = \frac{1}{2b_0} \frac{1}{\log(Q^2 / \Lambda^2)} \left[a_{qq}^n M_f^n(CI) + \frac{1+(-)^n}{2} a_{qG}^n M_G^n \right]$$

3) Fitting of experimental data

K.F. Liu, PRD (2000)

$$\bar{u} - \bar{d} \xrightarrow{x \rightarrow 0} x^{-1/2} \quad \text{O.K.}$$

But $\bar{u} + \bar{d} \propto \bar{s}$ is not correct.

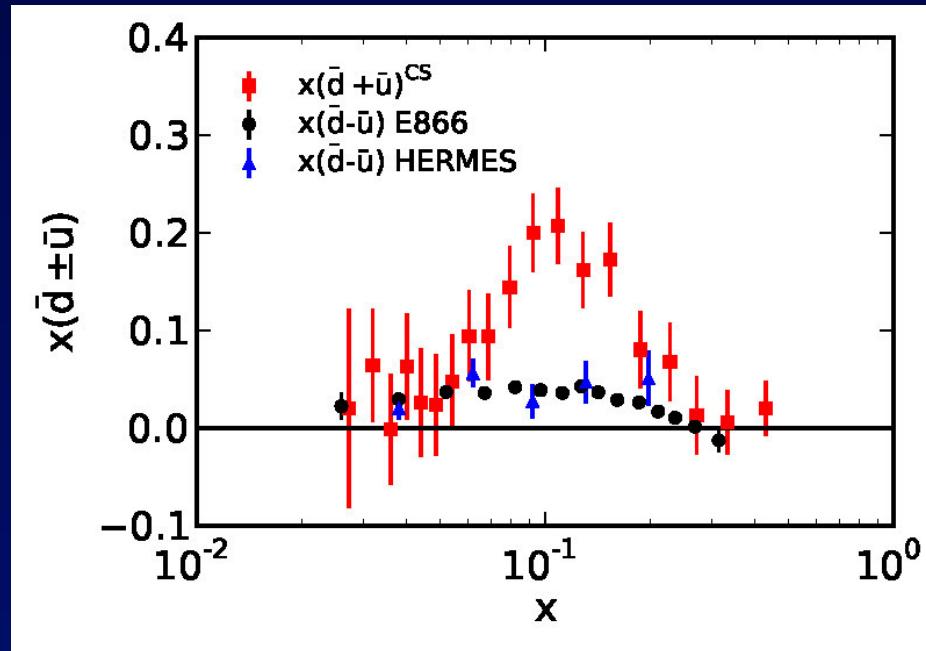
A better fit $\frac{\bar{u}(x) + \bar{d}(x)}{2} = f \bar{s}(x) + cs(x), \quad f \approx 1$

where $cs(x) \xrightarrow{x \rightarrow 0} x^{-1/2}$ like in $\bar{u}(x) - \bar{d}(x)$

4) Unlike DS, CS evolves the same way as the valence.

How to Extract Connected Sea Partons ?

K.F. Liu, W.C. Chang, H.Y. Cheng,
J.C. Peng, PRL 109, 252002 (2012)



$Q^2 = 2.5 \text{ GeV}^2$

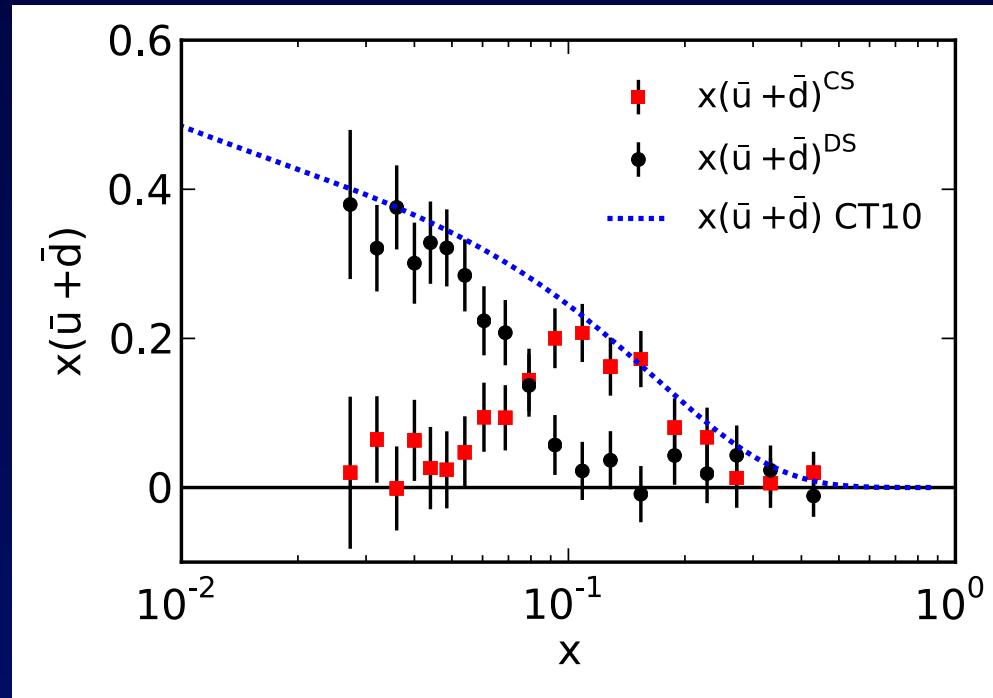
$$x(\bar{d} + \bar{u})_{CS}(x) = x(\bar{d} + \bar{u})(x) - \frac{1}{R} x(s + \bar{s})(x);$$

CT10

lattice

expt

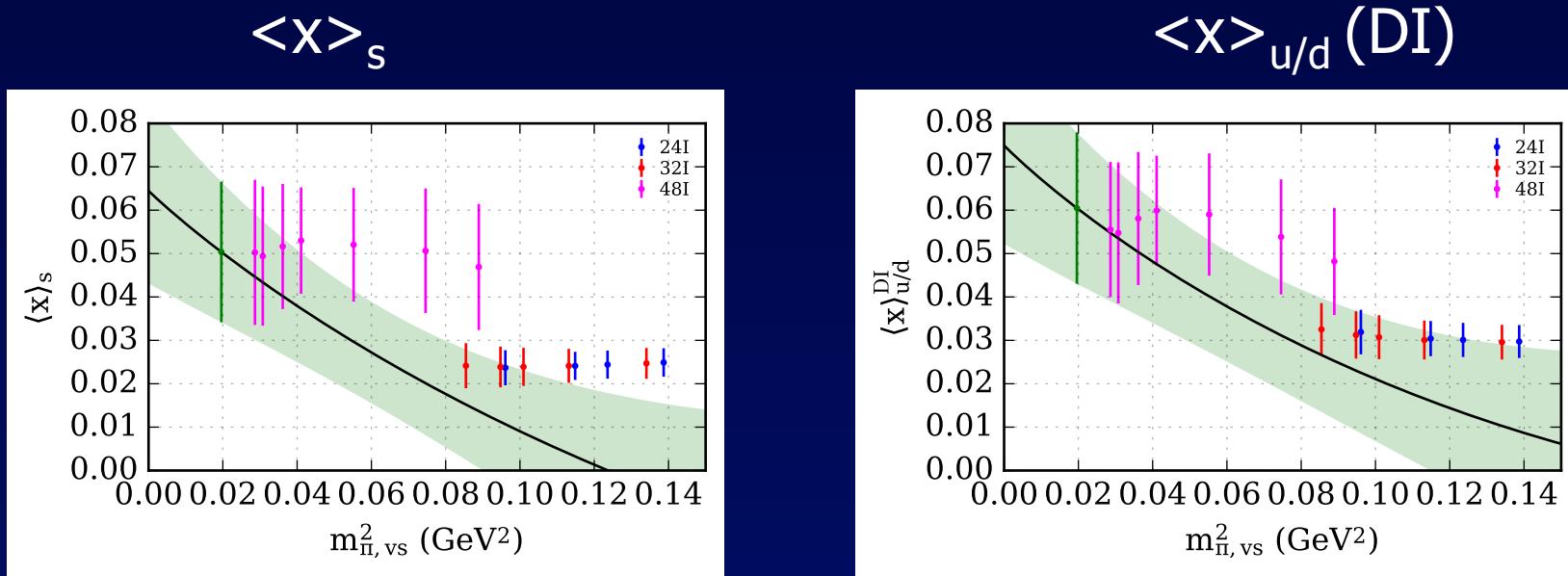
$$R = \frac{\langle x \rangle_s}{\langle x \rangle_u(DI)} (\text{lattice}) : 0.857$$



$$q_V, q_{CS}, \bar{q}_{CS} \sim_{x \rightarrow 0} x^{-\alpha_R} (x^{-1/2})$$

$$q_{DS}, \bar{q}_{DS} \sim_{x \rightarrow 0} x^{-1}$$

Lattice input to global fitting of PDF



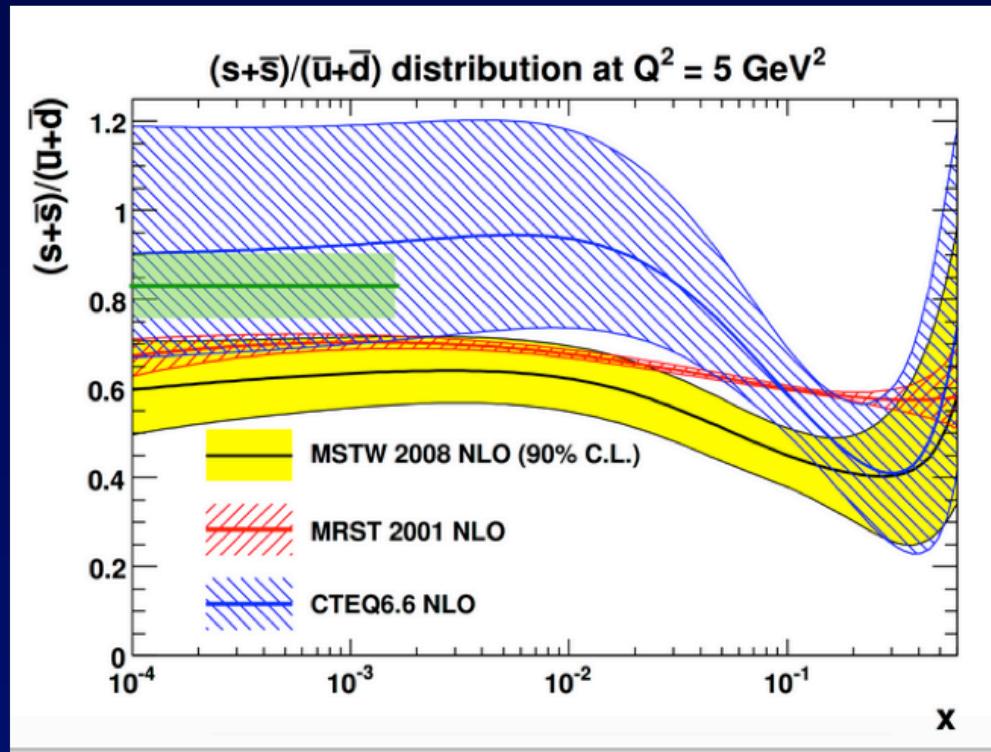
Lattice calculation with overlap fermion on 3 lattices including on at sea $m_\pi \sim 140$ MeV (Mingyang Sun, χ QCD Collaboration)

$$\text{Data} = a + b m_{\pi, vv}^2 + c m_{\pi, vs}^2 + d m_{\pi, vs}^3 + e a^2 + f e^{-m_{\pi, vv} L}$$

$$\langle x \rangle_s = 0.050(16), \langle x \rangle_{u/d} (\text{DI}) = 0.060(17)$$

$$\langle x \rangle_s / \langle x \rangle_{u/d} (\text{DI}) = 0.83(7) \quad Q^2 = 2 \text{ GeV}^2$$

Lattice input to global fitting of PDF



$$\langle x \rangle_s / \langle x \rangle_{u/d}(\text{DI}) 0.83(7) \quad Q^2=2 \text{ GeV}^2$$

Evolution Equations

S. Moch et al., hep/0403192, 0404111
A. Cafarella et al., 0803.0462

NNLO

$$\begin{aligned} dq_i / dt &= \sum_k (P_{ik} \otimes q_k + P_{i\bar{k}} \otimes q_{\bar{k}}) + P_{ig} \otimes g; \\ d\bar{q}_i / dt &= \sum_k (P_{\bar{i}k} \otimes q_k + P_{\bar{i}\bar{k}} \otimes q_{\bar{k}}) + P_{\bar{i}g} \otimes g; \\ dg / dt &= \sum_k (P_{gk} \otimes q_k + P_{g\bar{k}} \otimes q_{\bar{k}}) + P_{gg} \otimes g. \end{aligned}$$



$$dq_i^- / dt = P_{qq}^- \otimes q_i^- + \frac{P_{ns}^s}{N_f} \otimes \Sigma_v;$$

$$\text{where } q_i^- \equiv q_i - \bar{q}_i, \quad \Sigma_v \equiv \sum_k (q_k - \bar{q}_k),$$

$$\text{and } P_{ns}^s \sim O(\alpha_s^3)$$

Valence u can evolve into valence d ?

Note: $q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$

Evolution equations separating CS from the DS partons

$$dq_i^{v+cs} / dt = P_{ii}^c \otimes q_i^{v+cs} + P_{i\bar{i}}^c \otimes \bar{q}_i^{cs};$$

$$d\bar{q}_i^{cs} / dt = P_{i\bar{i}}^c \otimes \bar{q}_i^{cs} + P_{\bar{i}\bar{i}}^c \otimes q_i^{v+cs};$$

$$dq_i^{ds} / dt = \sum_k (P_{ik}^{cd} \otimes q_k^{ds} + P_{i\bar{k}}^{cd} \otimes \bar{q}_k^{ds} + P_{ik}^d \otimes q_k^{v+cs} + P_{i\bar{k}}^d \otimes \bar{q}_k^{cs}) + P_{ig} \otimes g;$$

$$d\bar{q}_i^{ds} / dt = \sum_k (P_{i\bar{k}}^{cd} \otimes \bar{q}_k^{ds} + P_{\bar{i}k}^{cd} \otimes q_k^{ds} + P_{i\bar{k}}^d \otimes q_k^{v+cs} + P_{\bar{i}k}^d \otimes \bar{q}_k^{cs}) + P_{ig} \otimes g;$$

$$dg / dt = \sum_k [P_{gk} \otimes (q_k^{v+cs} + q_k^{ds}) + P_{g\bar{k}} \otimes (\bar{q}_k^{cs} + \bar{q}_k^{ds}) + P_{gg} \otimes g].$$

Comments

- CS and DS are explicitly separated, leading to more equations (11 vs 7) which can accommodate $s \neq \bar{s}$, $u^{ds} \neq \bar{u}^{ds}$
- There is no flavor-changing evolution of the valence partons.

$$dq_i^- / dt = P_{qq}^- \otimes q_i^- + P_{ds}^- \otimes \sum_k (q_k - \bar{q}_k);$$

is the sum of two equations

$$dq_i^v / dt = P_{qq}^- \otimes q_i^v, \quad q^v \equiv q^{v+cs} - \bar{q}^{cs}$$

$$d(q_i^{ds} - \bar{q}_i^{ds}) / dt = \sum_k P_{ik}^{cd-} \otimes (q_k^{ds} - \bar{q}_k^{ds}) + \sum_k P_{ds}^{d-} \otimes q_k^v$$

- Once the CS is separated at one Q^2 , it will remain separated at other Q^2 .
- Gluons can split into DS, but not to valence and CS.
- It is necessary to separate out CS from DS when quark and antiquark annihilation (higher twist) is included in the evolution eqs. (Annihilation involves only DS.)

Comments

- The connected sea partons (CSP) found in path-integral formulation are extracted by combining PDF, experimental data and ratio of lattice matrix elements.
- It would be better to have separate evolution equations for the CSP and DSP. The separation will remain at different Q^2 .
- This way one can relate and compare the lattice calculation of moments in the CI and DI to the corresponding moments from PDF.
- It will become necessary to separate out CSP and DSP in evolution equations when quark and antiquark annihilation is involved.

Improved Maximum Entropy Method

- Inverse problem

$$D(\tau) = \int K(\tau, v) \rho(v) dv,$$

$$D(\tau) = \tilde{W}_{\mu\nu}(\tau), \quad K(\tau, v) = e^{-v\tau}, \quad \rho(v) = W_{\mu\nu}(q^2, v)$$

- Bayes' theorem

$$P[\rho | D] = \frac{P[D | \rho] P[\rho]}{P[D]}$$

- Maximum entropy method: find $\rho(v)$ from

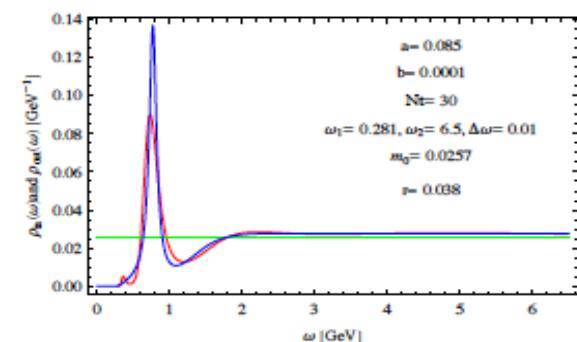
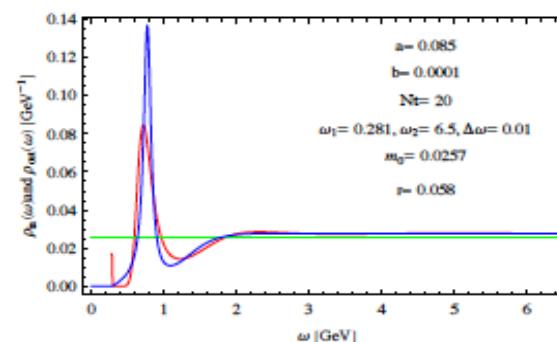
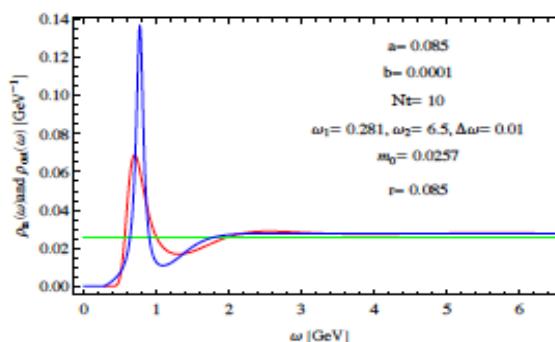
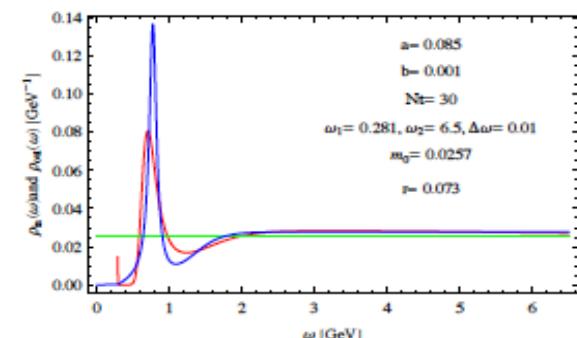
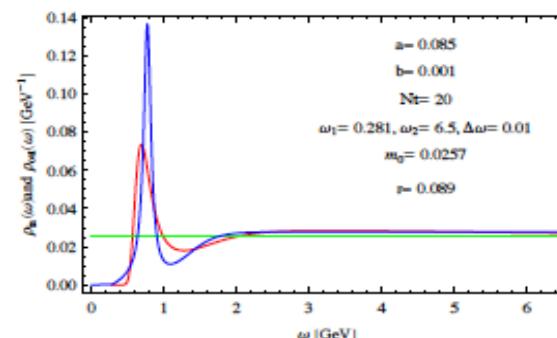
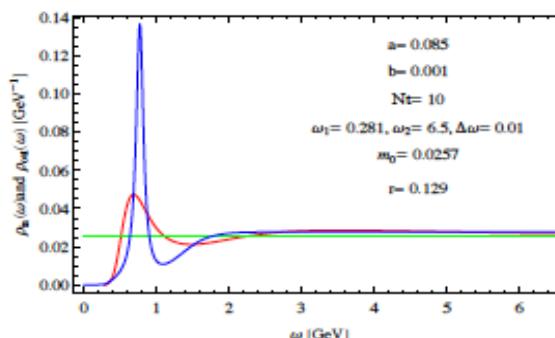
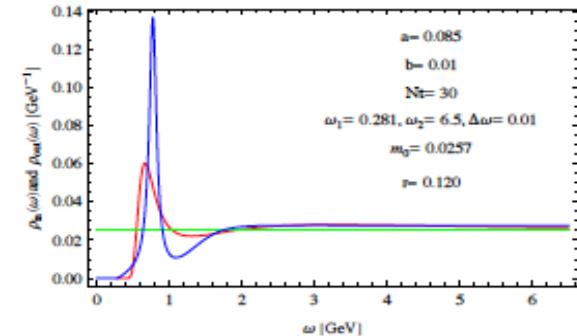
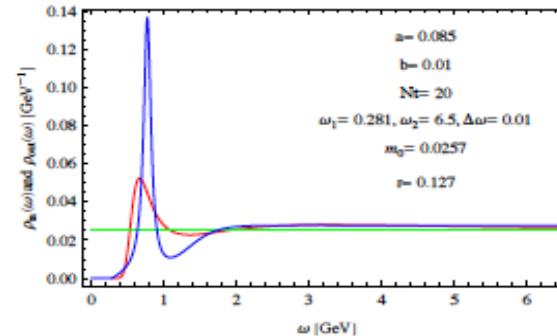
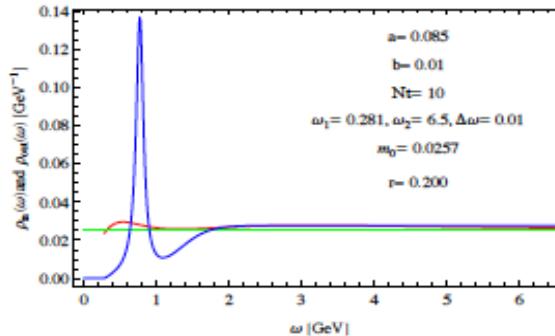
$$\frac{\partial P[\rho | D]}{\partial \rho} = 0$$

- Improved MEM (Burnier and Rothkpf, PRL 111, 182003 (2013))

$$P[\rho | D] \propto e^{\alpha S - L - \gamma(L - N_\tau)^2}, \quad L = \frac{\chi^2}{2}$$

$$S = \int dv \left[1 - \frac{\rho(v)}{m(v)} - \ln \left(\frac{\rho(v)}{m(v)} \right) \right]$$

Reconstruction of realistic data



Frank X. Lee
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Numerical Challenges

- Lattice calculation of the hadronic tensor – no renormalization, continuum and chiral limits, direct comparison with expts \rightarrow PDF.

- Bjorken x
$$x = \frac{Q^2}{2 p \cdot q} = \frac{\vec{q}^2 - v^2}{2(vE_p - \vec{p} \cdot \vec{q})}$$

- Range of x : $Q^2 = 2 \text{ GeV}^2$
 $-\vec{q} \parallel \vec{p} \quad |\vec{p}| = 3 \text{ GeV}, |\vec{q}| = 3 \text{ GeV}, \Rightarrow x = 0.058$

$$\vec{p} = 0, |\vec{q}| = 2 \text{ GeV} \Rightarrow x = 0.75$$

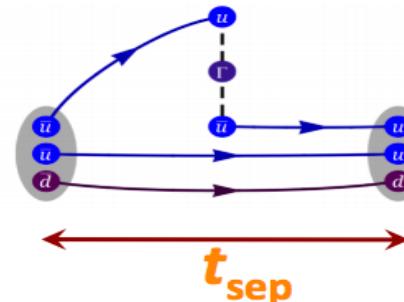
Large Momentum Approach

X. Ji, PRL, 110, 262002 (2013)

§ Take the large- P_z limit:

$$q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P | \bar{\psi}(z) \gamma^z \exp \left(-ig \int_0^z dz' A^z(z') \right) \psi(0) | P \rangle + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

↑ ↑ ↑
 $x = k^z/P^z$ Lattice z coordinate Nucleon momentum $P^\mu = \{P^0, 0, 0, P^z\}$



Product of lattice gauge links

- ❖ At $P^z \rightarrow \infty$ limit, twist-2 parton distribution is recovered
- ❖ For finite P^z , corrections are needed

Theoretical Issues

- Relatively simple numerically (H.W. Lin et al., 1402.1462; C. Alexandrou et al., 1504.07455)
- Renormalization of quasi-distribution (LaMET)

$$\tilde{q}(x, \mu^2, P_z) = \int_0^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y, \mu^2) + O\left(\frac{\Lambda^2}{P_z^2}, \frac{M^2}{P_z^2}\right)$$

- Perturbative and non-perturbative lattice renormalization
- Linear divergence of the Wilson line
(X. Xiong, X. Ji, Z.H. Zhang, Y. Zhao, 1310.7471;
T. Ishikawa, Y.Q. Ma, J.W. Qiu, S. Yoshida, 1609.02018)

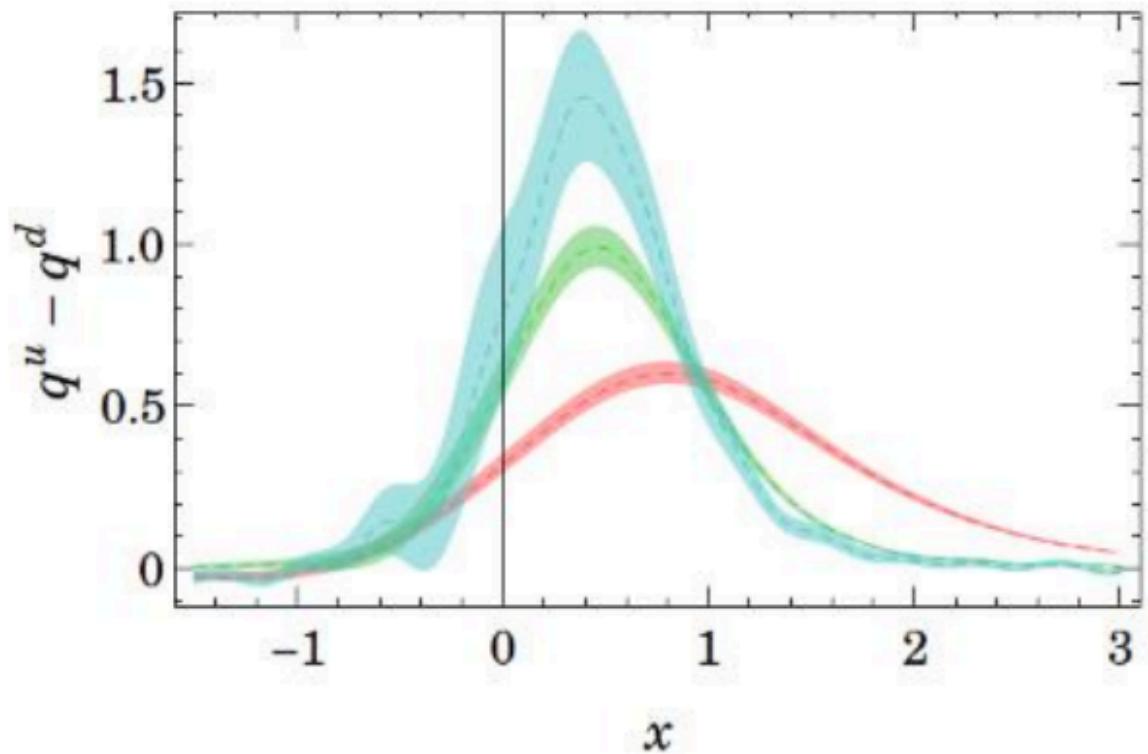
- How large P_z needs to be?

Quasi-PDF $u(x) - d(x)$

§ Do the integral

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle$$

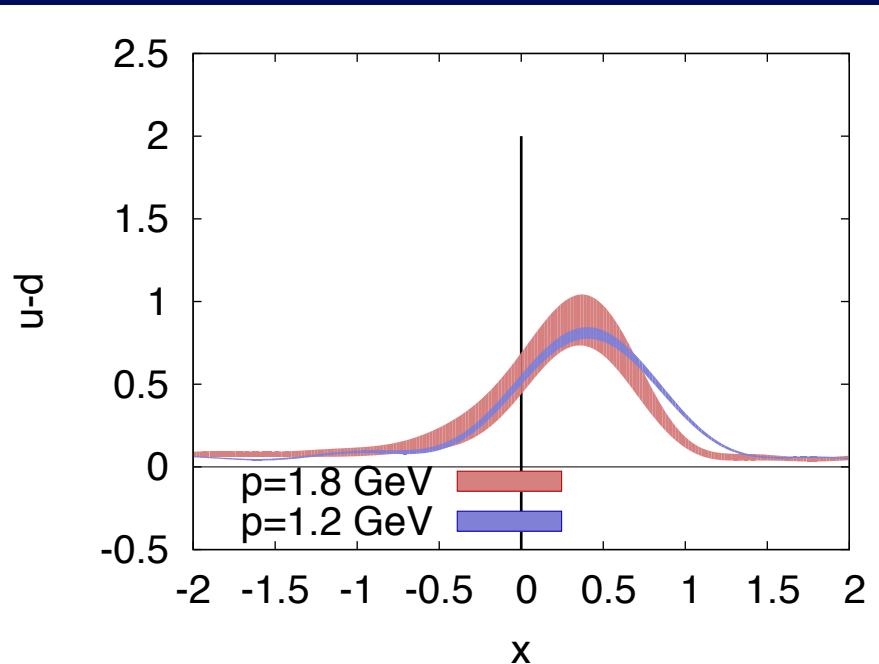
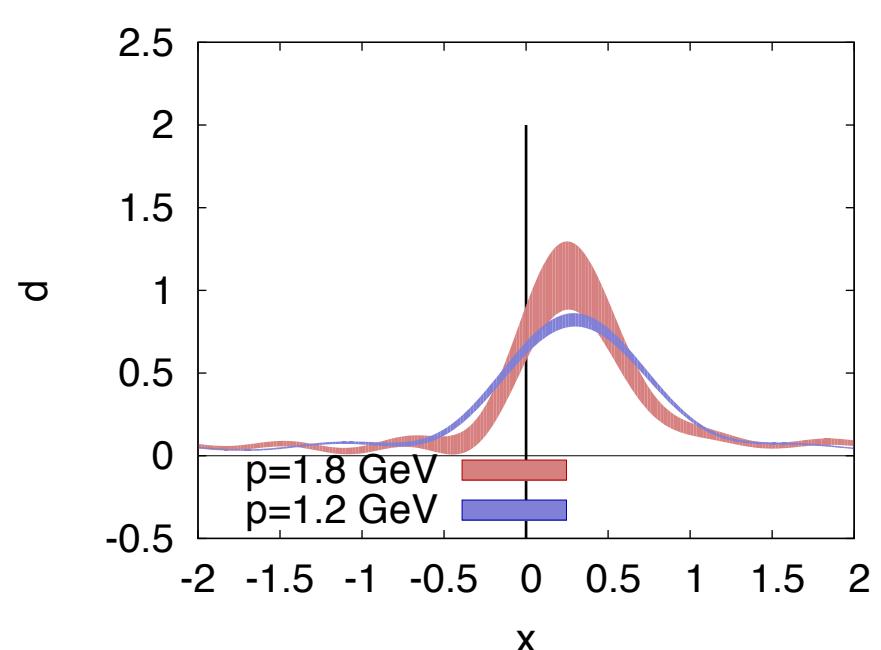
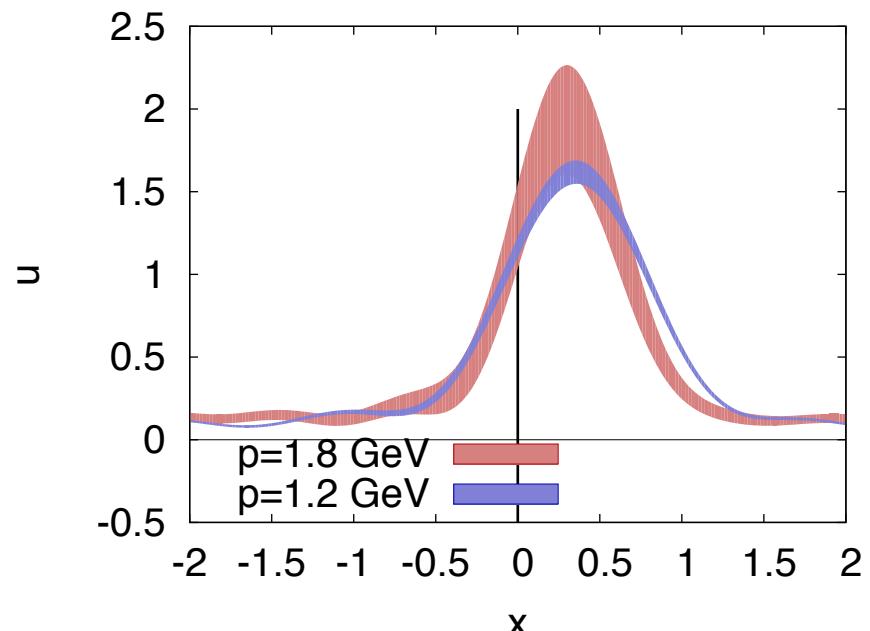
$$P_z \in \{0.43, 0.86, 1.29\} \text{ GeV}$$



Uncorrected bare
lattice results

$$x = k_z/P_z$$

Distribution should
sharper as P_z increases
Artifacts due to finite P_z
on the lattice

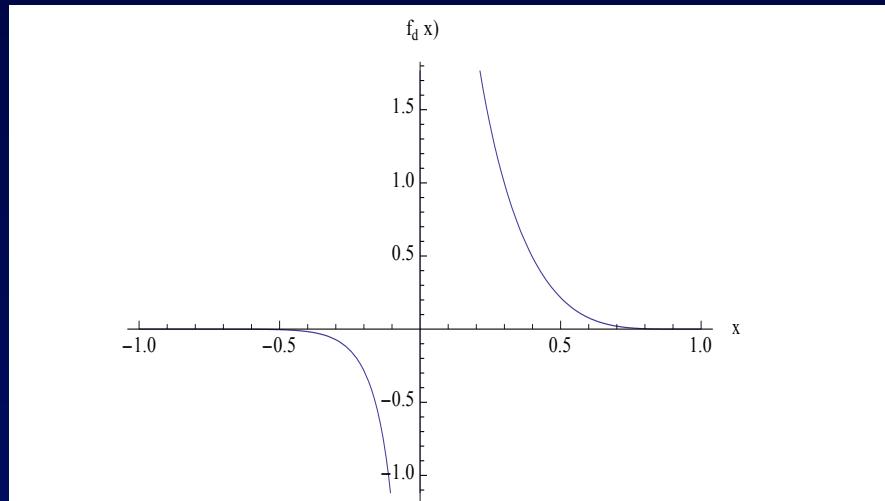


$$\bar{q}(x) = -f(-x)$$

$32^3 \times 64$ lattice at $a = 0.06 \text{ fm}$
 Clover on DWF configurations

$m_\pi(\text{val}) = 500 \text{ MeV}, m_\pi(\text{sea}) = 400 \text{ MeV}$

- Negative $\bar{q}(x)$ puzzle



d and \bar{d} from CTEQ6 (JW Chen)

$$\bar{d}(|x|) = -d(-|x|)$$

present $P_z \sim 1$ GeV

H.W. Lin, 1402.1462

C. Alexandrou, 1504.07455

- Larger P_z ? (How large)
- Lattice scale ($a^{-1} \sim 2$ GeV) too small?
- Range of x limited?

Summary

- Formulation of the hadronic tensor in Euclidean path-integral has revealed the connected sea parton (CSP) dof.
- It takes experiments, lattice calculation and global fitting of PDF to extract CSP.
- It is better to have CSP and DSP parton separated in evolution. This would facilitate comparison with lattice calculation of moments.
- Lattice calculation of hadronic tensor is numerically tough, but theoretically interpretation is relatively easy. No renormalization is needed and it can be calculated in the rest frame.
- Progress made with large momentum quasi-PDF. Both hadronic tensor and quasi-PDF approaches should be pursued and checked with experiments.

